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[This question paper contains 4 printed pages]

Your Roll No. :

Sl. No. of Q. Paper : **6635** **HC**

Unique Paper Code : 32351601

Name of the Course : **B.Sc.(Hons.)**
Mathematics

Name of the Paper : Complex Analysis

Semester : VI



Time : 3 Hours **Maximum Marks : 75**

Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
 - (b) All questions are compulsory.
 - (c) Question **NO.1** has been divided in **5** parts and each part is of **3** marks.
 - (d) Each question from **2** to **6** has **3** parts and each part is of **6** marks. Attempt any **two** parts from each question.
 - (e) Please, read the instructions carefully.
1. (a) Consider the complex function $f(z) = z^2$: Find images of x-axis (real axis) and y-axis (imaginary axis).
- (b) Show that the following functions are nowhere differentiable.
- (i) $f(z) = z + \bar{z}$,
 - (ii) $f(z) = e^y \cos x + ie^y \sin x$
- (c) Find all zeros of the function $\sin z$

P.T.O.

- (d) Using Cauchy Integral formula, evaluate, when
 $\gamma = C(0; 2) \equiv \{z \in \mathbb{C} : |z| = 2\},$

$$\int_{\gamma} \frac{z^3 + 5}{z - i} dz$$

- (e) Does there exist a function $f : D \rightarrow \mathbb{C}$ such that

$$f\left(\frac{1}{n}\right) = \frac{n}{n+1} \text{ for all } n \in \mathbb{N}, \text{ where } D \text{ is unit disc in the complex plane } \mathbb{C}.$$

2. (a) Prove that :

$$\lim_{z \rightarrow 0} \frac{\bar{z}^2}{z} = 0$$

Show that

$$\lim_{z \rightarrow \infty} \frac{z^2}{(2z-1)^2} = \frac{1}{4}$$

- (b) Show that the given functions are continuous everywhere on $\mathbb{C}$.

(i) The conjugation map $f(z) = \bar{z}$.

(ii) The affine map $f(z) = az + b$; where $a, b \in \mathbb{C}$ and $a \neq 0$.

- (c) Show that if a function f is continuous throughout a closed and bounded region D , there exists a nonnegative real number M such that $|f(z)| \leq M$ for all points $z \in D$, where equality holds for at least one such z :

3. (a) Prove that f defined by

$$f(z) = \begin{cases} \frac{z^5}{|z|^4} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

satisfies the C-R equations at $z = 0$ but is not differentiable there.

- (b) Suppose that f is an analytic function in a domain D (non-empty open connected set). Show that f will be a constant function if the following conditions are satisfied in D :

(i) $f'(z) = 0 \quad \forall z \in D;$

(ii) $|f(z)|$ constant in D .

- (c) Find numbers $z = x + iy$ such that $e^z = 1 + i$.

4. (a) Without evaluating the integral, show that

$$\left| \int_C \frac{dz}{z^2 - 1} \right| \leq \frac{\pi}{3}$$

where C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant.

- (b) If C is positively oriented contour $|z| = 1$ and $f(z) =$

e^z , evaluate $\int_C \frac{e^z}{z^4} dz$.

- (c) Let z_1 and z_2 be two distinct complex numbers that lie interior to the simple closed contour C . Show that :

$$\int_C (z - z_1)^{-1} (z - z_2)^{-1} dz = 0$$

5. (a) Define an entire function. Show that if a function is entire and bounded in the complex plane, then it is constant throughout the plane.

- (b) Give two Laurent series expansions in powers of z

for the function $f(z) = \frac{1}{z^2(1-z)}$ and specify the regions in which those expansions are valid.

- (c) Find the Maclaurin series expansion of the function $\sin z$.

6. (a) Find the nature of the singularities of the following functions

$$(i) f_1(z) = \frac{z - \sin z}{z^4} \quad (ii) f_3(z) = \frac{1 - e^z}{1 + e^z}$$

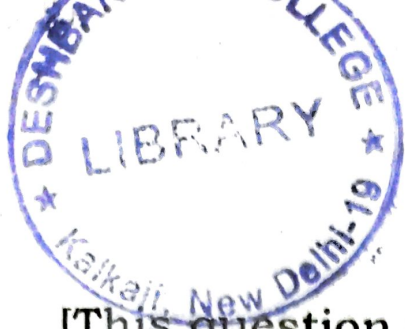
- (b) State Cauchy's Residue Theorem. Use it to

evaluate the integral $\int_C \frac{5z-2}{z(z-1)} dz$

where C is the circle $|z| = 2$, described counterclockwise.

- (c) Show that :

$$\int_0^{2\pi} \frac{\cos 2\theta d\theta}{5 - 4\cos\theta} = \frac{\pi}{6}$$



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Your Roll No. :

Sl. No. of Q. Paper : **6636** **HC**

Unique Paper Code : 32351602

Name of the Course : **B.Sc.(Hons.)Mathematics**

Name of the Paper : Ring Theory and Linear
Algebra-II

Semester : VI

Time : 3 Hours **Maximum Marks : 75**

Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
- (b) Attempt any **two** parts from each of the question.

1. (a) State and prove Division Algorithm for $F[x]$ where F is a field.

(b) Let p be a prime and $f(x) \in \mathbb{Z}_p[x]$ be irreducible over $\mathbb{Z}_p$ such that $\deg f(x) = n$, prove that $\frac{\mathbb{Z}_p[x]}{\langle f(x) \rangle}$ is a field with p^n elements.

P.T.O.

- (c) In $\mathbb{Z}[\sqrt{-5}]$, prove that $1 + 3\sqrt{-5}$ is irreducible but not prime. 6, 6, 6

2. (a) Prove that a polynomial of degree n over a field F has at most n zeros, counting multiplicity. Give an example to show that the result does not hold if F is not a field.
- (b) Let F be a field and $p(x) \in F[x]$. Then prove that $\langle p(x) \rangle$ is a maximal ideal in $F[x]$ if and only if $p(x)$ is irreducible over F .
- (c) Prove that the ring of Gaussian integers $\mathbb{Z}(i)$ is a Euclidean domain. 6.5, 6.5, 6.5

3. (a) Let $V = P_1(1+R)$ and for $p(x) \in V$, define ,

$$f_1, f_2 \in V \quad \text{by} \quad f_1(p(x)) = \int_0^1 p(t) dt \quad \text{and}$$

$$f_2(p(x)) = \int_0^2 p(t) dt$$

Prove that f_1, f_2 is a basis for V^* and find a basis for V for which it is the dual basis.

6

- (b) Prove that a linear operator T on a finite dimensional Vector space V is diagonalizable if and only if there exists an ordered basis β for V consisting of eigen vectors of T .

- (c) Let $A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$ Show that A is diagonalizable and find a 2×2 matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

6, 6, 6

4. (a) Let T be a linear operator on a finite dimensional vector space V and W be a T invariant subspace of V . Prove that the characteristic polynomial of T_W divides the characteristic polynomial of T .
- (b) Let T be a linear operator on an n dimensional vector space V such that V is a T cyclic subspace of itself. Prove that $f(t)$, the characteristic polynomial of T and $p(t)$ the minimal polynomial of T have the same degree.
- (c) Let T be a linear operator $P_2(\mathbb{R})$ on such that $T(g(x)) = g'(x) + 2g(x)$. Find minimal polynomial of T . Hence, also determine if T is diagonalizable.

6.5, 6.5, 6.5

5. (a) State and prove Cauchy Schwarz inequality in an inner product space V over F and use it to prove triangle inequality in V .

- (b) Apply the Gram-Schmidt process to the given subset S of the inner product space V to obtain an orthonormal basis β for $\text{span}(S)$ and compute the Fourier coefficients of the given vector x relative to β .

$$V = \mathbb{R}^3, S = \{(1,1,1), (0,1,1), (0,0,1)\} \text{ and } x = (1,0,1)$$

- (c) Let V be a finite dimensional inner product space and let T be a linear operator on V . Show that there exists a unique linear operator $T^*: V \rightarrow V$ such that

$$\langle T(x), Y \rangle = \langle x, T^*y \rangle \text{ for all } x, y \in V.$$

6.5, 6.5, 6.5

6. (a) For the set of data $\{(-3,9), (-2,6), (0,2), (1,1)\}$, use the least squares approximation to find the best fit with a linear function. Also compute the error E .

- (b) State and prove Schur Theorem.

- (c) Let V be an inner product space and let T be a linear operator on V . Then prove that T is an orthogonal projection if and only if T has an adjoint T^* and $T^2 = T = T^*$.

6, 6, 6

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Your Roll No. :

Sl. No. of Q. Paper : **9488** **HC**

Unique Paper Code : 32357607

Name of the Course : **B.Sc.(Hons.)Mathematics** :
DSE-3

Name of the Paper : Probability Theory &
Statistics

Semester : VI

Time : 3 Hours **Maximum Marks : 75**

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
- In all there are **six** questions.
- Question No.1 is compulsory, and it contains **seven** parts of **3** marks each, out of which any **five** parts are to be attempted .
- In Question No.2 to **6**, attempt any **two** parts . Each part carries **6** marks.
- Use of scientific calculator is allowed.

P.T.O.



1. (i) The distribution function of a random variable is

$$F(x) = \begin{cases} 0, & x < -2 \\ \frac{x+4}{8} & \text{if } -2 \leq x < 2 \\ x^2 & \text{if } x \geq 2. \end{cases}$$

Obtain $P(X = -2)$, $P(X = 2)$ and $P(0 < X \leq 2)$.

- (ii) Find the moment generating function (mgf) of a random variable X having the probability density function (pdf):

$$f(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty.$$

- (iii) A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

- (iv) If X is a random variable having pmf

$$p(x) = \begin{cases} 1/8, & x = -1 \\ 6/8, & x = 0 \\ 1/8, & x = 1. \end{cases}$$

Use Chebyshev's inequality to obtain an upper bound of $P[|X| \geq 1]$.

- (v) If the joint distribution of X and Y is given by

$$F(x, y) = \begin{cases} (1 - e^{-x})(1 - e^{-y}), & x > 0, y > 0 \\ 0, & \text{elsewhere,} \end{cases}$$

Find $P(1 < X \leq 3, 1 < Y \leq 3)$.

- (vi) If the probability is 0.75 that an applicant for driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try?
- (vii) If the joint mgf of two random variables X_1 and X_2 is given by

$$M(t_1, t_2) = e^{\mu_1 t_1 + \mu_2 t_2 + \frac{1}{2}(t_1^2 \sigma_1^2 + t_2^2 \sigma_2^2) + \rho t_1 t_2 \sigma_1 \sigma_2}$$

Where $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$ are constants, then show that $\text{Cov}(X, Y) = \rho \sigma_1 \sigma_2$.

2. (a) (i) Prove that for any random variable,

$$P(X = x) = F_x(x) - F_x(x-)$$

for all $x \in R$, where $F_x(x-) = \lim_{z \uparrow x} F_x(z)$



- (ii) There are 5 red chips and 3 blue chips in a bowl. The red chips are numbered 1,2,3,4,5 respectively and the blue chips are numbered 1,2,3 respectively. If 2 chips are to be drawn at random and without replacement, find the probability that these chips will have either the same number or the same colour.

- (b) Let X have the pdf

$$f(x) = \frac{x^2}{18}, -3 < x < 3,$$

= zero elsewhere.

- (i) Find the cdf of X
 (ii) Find $P(|X| < 1)$ and $(X^2 < 9)$
 (c) (i) Find the characteristic function of Poisson distribution with parameter λ .
 (ii) Find the median of the distribution with probability mass function

$$p(x) = \begin{cases} \frac{\binom{4}{x} \binom{4-x}{4-x}}{\binom{4}{x} \binom{4-x}{4-x}} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{4-x}, & x = 0, 1, 2, 3, 4 \\ 0 & \text{elsewhere} \end{cases}$$

3. (a) Let X have the uniform continuous distribution with parameters α and β .

Find the mgf, the mean, and the variance of this distribution.

- (b) Let a random variable X have the Poisson distribution with parameter λ . Then show that as $\lambda \rightarrow \infty$, the moment generating function of $Z = \frac{X - \lambda}{\sqrt{\lambda}}$ approaches the moment generating function of standard normal distribution.
- (c) Prove that normal distribution is a symmetrical distribution.

4. (a) Let (X_1, X_2) have the joint pmf as given below :

(x_1, x_2)	(0,0)	(0,1)	(0,2)	(1,1)	(1,2)	(2,2)
$p(x_1, x_2)$	1/12	2/12	1/12	3/12	4/12	1/12

Find $E(X_1), E(X_2), Var(X_1)$ and $Var(X_2)$.

Is $E(X_1 X_2) = E(X_1)E(X_2)$?

- (b) Let (X_1, X_2) have the joint cdf $F(x_1, x_2)$ and let X_1 and X_2 have the marginal cdfs $F_1(x_1)$ and $F_2(x_2)$ respectively. Prove that X_1 and X_2 are independent if and only if

$$F(x_1, x_2) = F_1(x_1)F_2(x_2)$$

for all $(x_1, x_2) \in \mathbb{R}^2$.



- (c) Given the two random variables X and Y that have the joint density

$$f(x, y) = \begin{cases} 24xy, & x > 0, y > 0, x + y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the regression equation of X on Y .
Also compute $P(0 < X < 1/2 \mid Y = 5/8)$.

5. (a) Let X and Y have the joint pdf

$$f(x, y) = \begin{cases} e^{-y}, & 0 < x < y < \infty \\ 0, & \text{elsewhere.} \end{cases}$$

Find the joint mgf of X and Y and hence compute the correlation coefficient.

- (b) Let X and Y have the joint pdf

$$f(x, y) = \begin{cases} 3x, & 0 < y < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the conditional mean and variance of X given $Y = y$, $0 < y < 1$.

- (c) Define bivariate normal distribution for a random vector (X, Y) and find marginal density function of Y .

6. (a) Let $\{X_i\}$ be a sequence of independent, identically distributed random variables, each with mean μ and variance σ^2 . Then

show that the mgf of $\frac{X_1 + \dots + X_n}{\sqrt{n}}$ is

$\left(1 + \frac{t^2}{2n}\right)^n$, when n is large. Hence prove the central limit theorem.

- (b) (i) The lifetime of a special type of battery is a random variable with mean 40 hours and standard deviation 20 hours. A battery is used until it fails at which point it is replaced by a new one. Assuming a stockpile of 20 such batteries, the lifetimes of which are independent, approximate the probability that over 1100 hours of use can be obtained.

(Use $\Phi(1) = 0.8413$)



- (ii) Let the Markov chain consisting of the states 0,1,2,3 have the transition probability matrix

$$\begin{vmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

Determine which states are transient and which are recurrent.

- (c) If the probability density of X is given by

$$f(x) = 630 x^4 (1-x)^4 \text{ for } 0 < x < 1 \\ = 0 \text{ elsewhere}$$

find the probability that it will take on a value within two standard deviations of the mean and compare this probability with the lower bound provided by Chebyshev's theorem.

(4)

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Your Roll No. :

Sl. No. of Q. Paper : **9488-A HC**

Unique Paper Code : 32357610

Name of the Course : **B.Sc.(Hons.)
Mathematics : DSE-4**

Name of the Paper : Number Theory

Semester : VI

Time : 3 Hours

Maximum Marks : 75

Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
- (b) Attempt **all** questions selecting **eight** parts from section **I**, **five** parts each section **II** and **III**.

Section - I

(Attempt any **eight** parts)

$8 \times 2\frac{1}{2}$

(a) Show that for any integer a , $a^3 \equiv 0, 1 \text{ or } 6 \pmod{7}$.

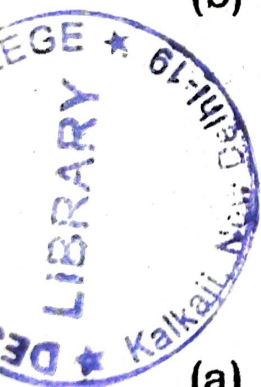
(b) Use Gauss Lemma to compute Legendre

symbol $\left(\frac{11}{23}\right)$.

(c) Use Quadratic Reciprocity law to compute

Legendre symbol $\left(\frac{1234}{4567}\right)$.

P.T.O.



- (d) Use Fermat theorem to test if 17 divides $13^{98}-1$.
- (e) Find the order of the integer 3 modulo 19.
- (f) Prove that if the integer n has r distinct odd prime factors, then $2^r \mid \phi(n)$.
- (g) For the Exponential cryptosystem (Pohling - Hellman system), with the public key $(41,7)$, compute the deciphering exponent.
- (h) Encrypt the message I LOVE DOING NUMBER THEORY using the linear cipher $C \equiv 2P+4 \pmod{26}$.
- (i) Find the number of primitive root of 2029.
- (j) Write a short note on Gauss and his work.
- (k) Define a multiplicative function. Compute $\tau(1245)$.
- (l) Prove that $\phi(2^n-1)$ is a multiple of n for any $n > 1$.

Section - II

(Attempt any **five** parts)

$5 \times 5 \frac{1}{2}$

1. Using Euclidean algorithm and theory of linear Diophantine equation, divide 100 into two summands such that one is divisible by 7 and other by 11.
2. State and prove Wilson's theorem.

3. Prove that the congruences $x \equiv a \pmod{n}$ and $x \equiv 3 \pmod{m}$ admit a simultaneous solution if and only if $\gcd(n, m) \mid (a-3)$; if a solution exists, confirm that it is unique modulo $\text{lcm}(n, m)$
4. Solve $x \equiv 5 \pmod{6}$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$.
5. Prove there are an infinite number of primes of the form $4n + 3$.
6. State Fermat's theorem and show, by means of an example, that the converse is not true,
7. Define a complete set of residues modulo p . Prove that if $\gcd(a, n) = 1$, then the integers $c, c + a, c + 2a, c + 3a, \dots, c + (n-1)a$ form a complete set of residues modulo n for any c .

Section - III

(Attempt any **five** questions)

$5 \times 5 \frac{1}{2}$

8. State and prove Quadratic Reciprocity Law.
9. Encrypt the plaintext message GOLD using RSA algorithm with key $(2561, 7)$.
10. Let p be an odd prime and $\gcd(a, p) = 1$, then show that a is a quadratic residue of p if and only if $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$.



11. Solve $3x^2 + 9x + 7 \equiv 0 \pmod{13}$.
12. If the integer a has the order k modulo n and $h > 0$, then show that a^h has order $\frac{k}{\gcd(h, k)}$ modulo n .
13. If p is an odd prime and $\gcd(a, p) = 1$, then show that the congruence $x^2 \equiv a \pmod{p^n}$, where $n \geq 1$ has a solution if and only if $\left(\frac{a}{p}\right) = 1$.
14. Let r be a primitive root of the odd prime p . Prove that if $p \equiv 3 \pmod{4}$, then $-r$ has order $\frac{p-1}{2}$ modulo p .
15. Prove that if p is prime number and $d \mid p-1$, then congruence $x^d - 1 \equiv 0 \pmod{p}$ has exactly d solutions.
16. If $p \neq 3$ is an odd prime, then show that

$$\left(\frac{3}{p}\right) = \begin{cases} 1, & \text{if } p \equiv \pm 1 \pmod{12} \\ -1, & \text{if } p \equiv \pm 5 \pmod{12} \end{cases}$$

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Roll No.

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S. No. of Question Paper : 9489A

Unique Paper Code : 32357611

HC

Name of the Paper : Linear Programming and Theory of Games

Name of the Course : B.Sc. (Hons.) Mathematics : DSE-4

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

All questions carry equal marks.

1. (a) Consider the following problem :

$$\text{Minimize } z = cx$$

Subject to

$$Ax = b$$

$$x \geq 0$$



P.T.O.

where A is an $m \times n$ matrix with rank m . Let x be the extreme point of the feasible region. Show that x is also a basic feasible solution of the system :

$$Ax = b, x \geq 0.$$

(b) Solve the following problem by simplex method :

$$\text{Maximize } z = x_1 - 2x_2 + x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 12$$

$$2x_1 + x_2 - x_3 \leq 6$$

$$-x_1 + 3x_2 \leq 9$$

$$x_1, x_2, x_3 \geq 0.$$

(c) Use simplex method to obtain the inverse of the following matrix :

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 9 \end{bmatrix}.$$

2. (a) Solve the following problem by two-phase method :

Maximize $z = 5x_1 - 2x_2 + x_3$

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Subject to :

$$2x_1 + 4x_2 + x_3 \leq 6$$

$$2x_1 + x_2 + 3x_3 \geq 2$$

$$x_1, x_2 \geq 0$$

x_3 is unrestricted.



- (b) Solve the following problem by the big-M method :

Minimize $z = 3x_1 + 2x_2 + 4x_3 + 8x_4$

Subject to

$$x_1 + 2x_2 + 5x_3 + 6x_4 \geq 8$$

$$-2x_1 + 5x_2 + 3x_3 - 5x_4 \leq 3$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

- (c) Find all the basic feasible solution of the equations :

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2.$$

3. (a) Show that the objective function value for any feasible solution to the minimization problem is always greater than or equal to the objective function value for any feasible solution to the maximization problem.

- (b) Write the dual of the primal problem given below :

$$\text{Min } z = x_1 + x_2 + x_3$$

Subject to

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 \leq 3$$

$$2x_2 - x_3 \geq 4$$

$$x_1, x_2 \geq 0 \quad x_3 \text{ is unrestricted.}$$

- (c) Use the graphical method to solve the dual of the following problem :

$$\text{Max } z = x_1 + 2x_2 + 6x_3$$

Subject to

$$3x_1 + x_2 + 2x_3 \leq 6$$

$$2x_1 + 3x_2 + x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0.$$



Further, use the complementary slackness theorem to find an optimal solution to the given problem from optimal solution of the dual problem.

- (a) Solve the following transportation problem :

	D₁	D₂	D₃	D₄	Availability
S₁	2	3	11	7	6
S₂	1	0	6	1	1
S₃	5	8	15	9	10
Requirement	7	5	3	2	

(b) Solve the following assignment problem :

	I	II	III	IV	V
1	3	8	2	10	3
2	8	7	2	9	7
3	6	4	2	7	5
4	8	4	2	3	5
5	9	10	6	9	10

(c) (i) Find minimax and maximin for the following game :

1	3	6
2	1	3
6	2	1

(ii) Define for theory of Games :

(I) Rectangular Game

(II) Pay off matrix

(III) Saddle Point.

5. (a) Solve the following game graphically :

	Player B			
Player A				
	$\begin{bmatrix} 2 & 2 & 3 & -2 \\ 4 & 3 & 2 & 6 \end{bmatrix}$			

- (b) Solve by principle of dominance, the following game :

	B_1	B_2	B_3
A_1	$\begin{bmatrix} 2 & -2 & 3 \end{bmatrix}$		
A_2	$\begin{bmatrix} -3 & 5 & -1 \end{bmatrix}$		
A_3	$\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$		



- (c) Give the linear programming formulation for player A and player B for the following game :

	Player B		
Player A			
	$\begin{bmatrix} 9 & 1 & 4 \\ 0 & 6 & 3 \\ 5 & 2 & 8 \end{bmatrix}$		